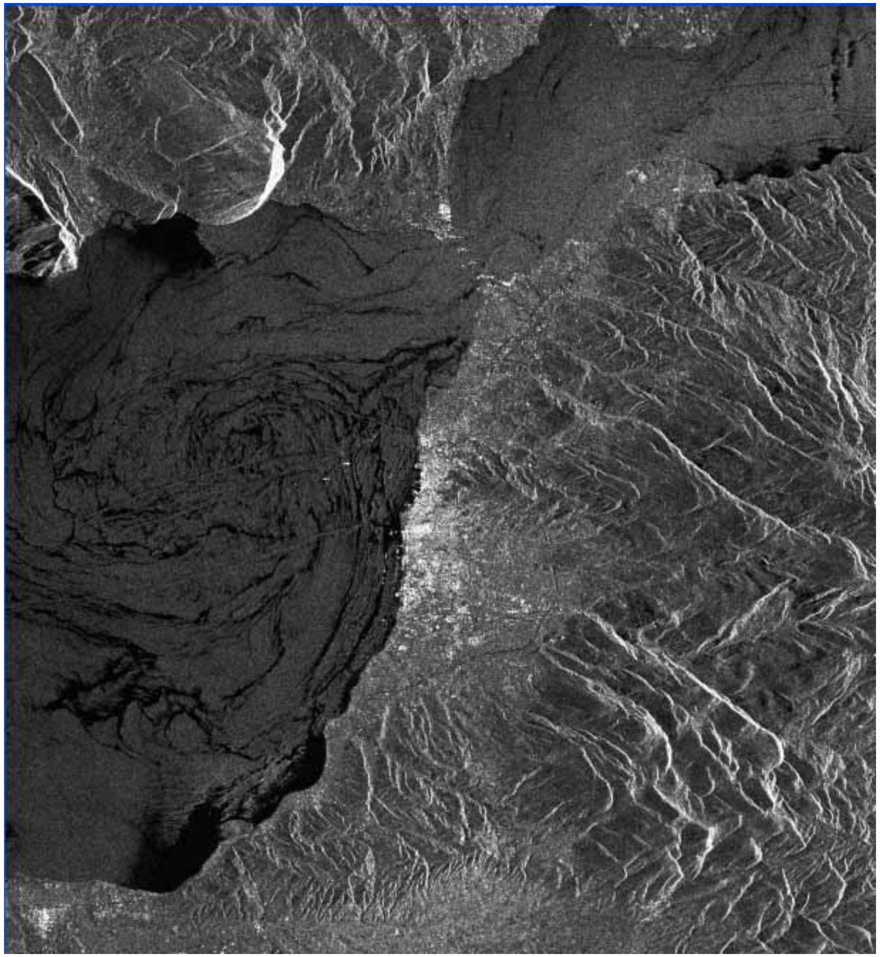
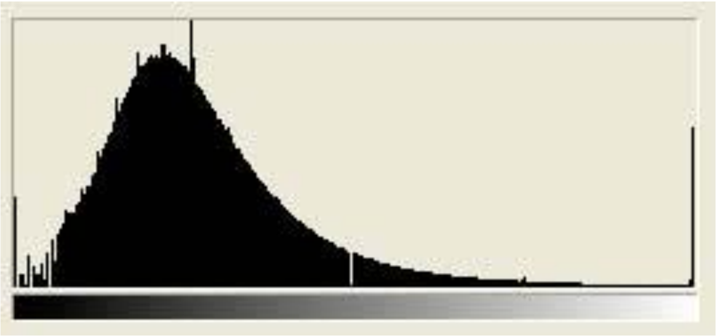
Signal Detection Basics - CFAR

- Types of noise (clutter) and signals (targets)
- Signal separation by comparison (threshold detection)
- Signal Statistics - Parameter estimation
- Threshold determination based on the required P_{fa}
- CFAR detectors design
- Detection Performance

Vassilis Anastassopoulos, Physics Department

Noise (clutter) and Signal (Target)





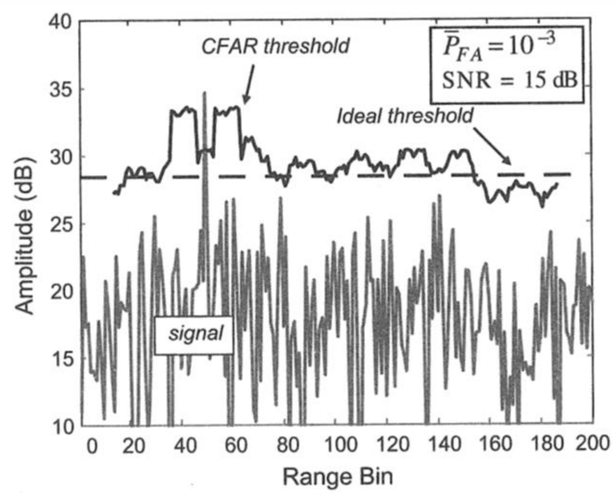
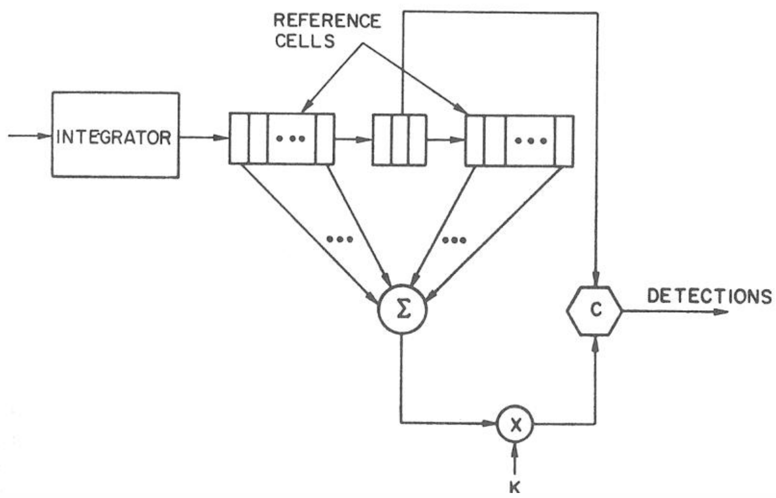


Figure 7.4 Example of cell-averaging CFAR threshold behavior.

Detection by comparison with a threshold



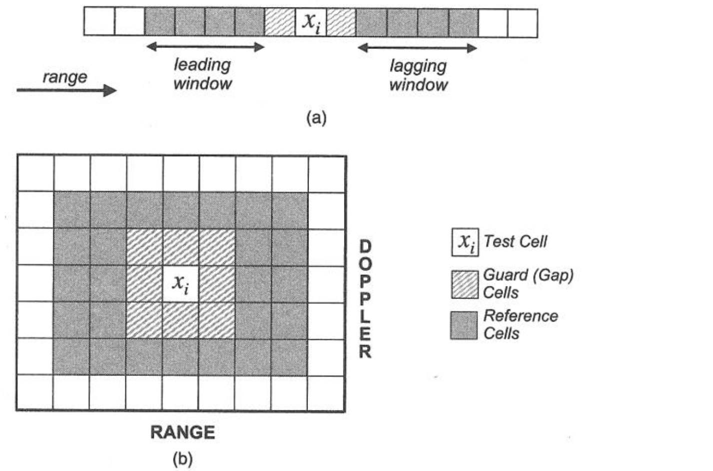
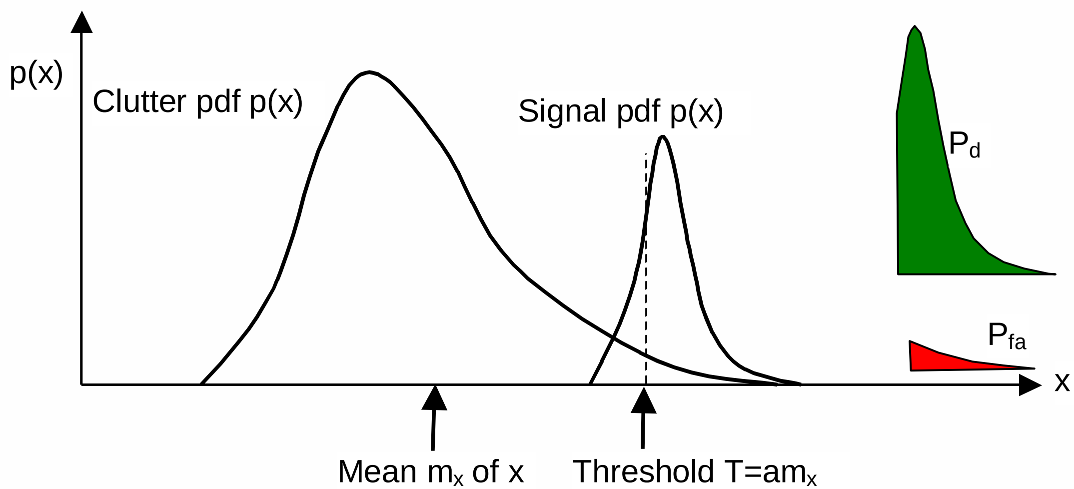
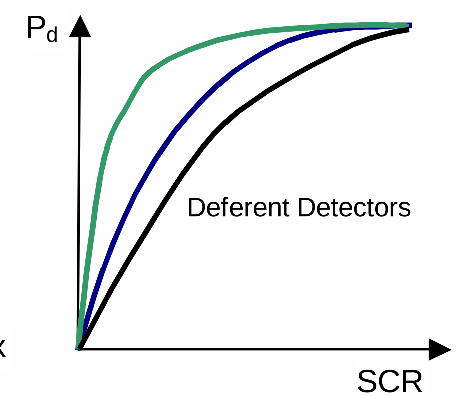
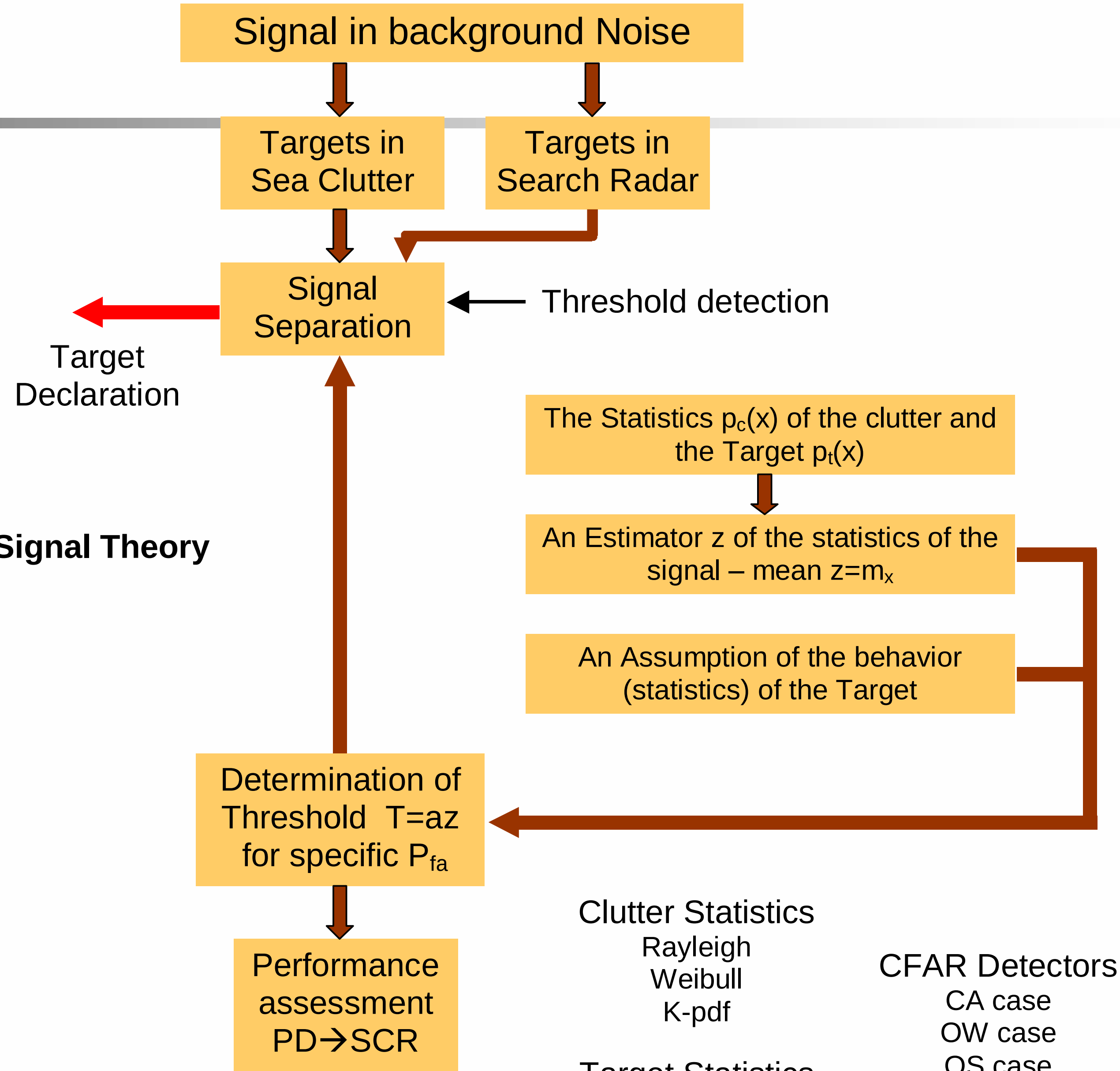


Figure 7.3 CFAR windows. (*a*) One-dimensional window for range-only processor. (*b*) Two-dimensional window for range-Doppler processor.





Receiver Operating Characteristics



Fundamentals of Radar Signal Theory
Mark Richards
McGraw-Hill
2005
Chapters 6 & 7

Clutter Statistics
Rayleigh
Weibull
K-pdf

Target Statistics
Swerling models

CFAR Detectors
CA case
OW case
OS case

Radar Detection as Hypothesis Testing

Hypothesis H_0: The measurement is a result of interference only **(null Hypothesis)**
Hypothesis H_1: The measurement is a combined result of interference and echoes from Targets

The signals are described statistically by means of their pdfs and thus we have to employ **Statistical Decision Theory.**

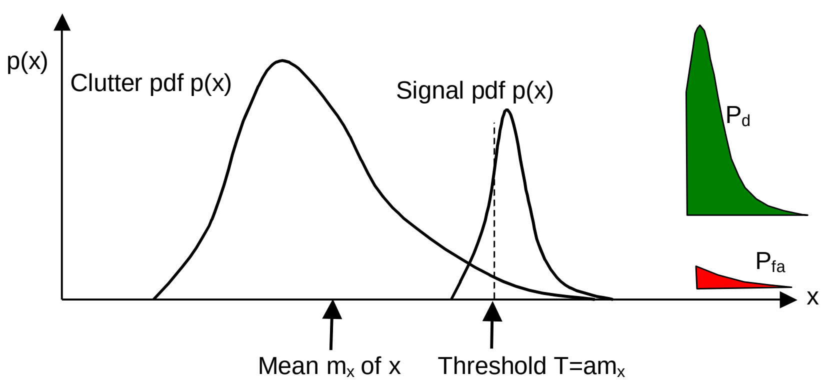
Actually we need the pdf of the Noise $p_c(x)$ and the pdf of the Target $p_t(x)$:

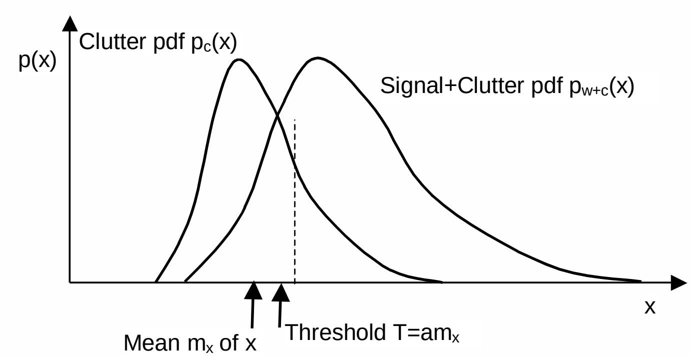
Clutter $\rightarrow$ x

Target $\rightarrow$ w

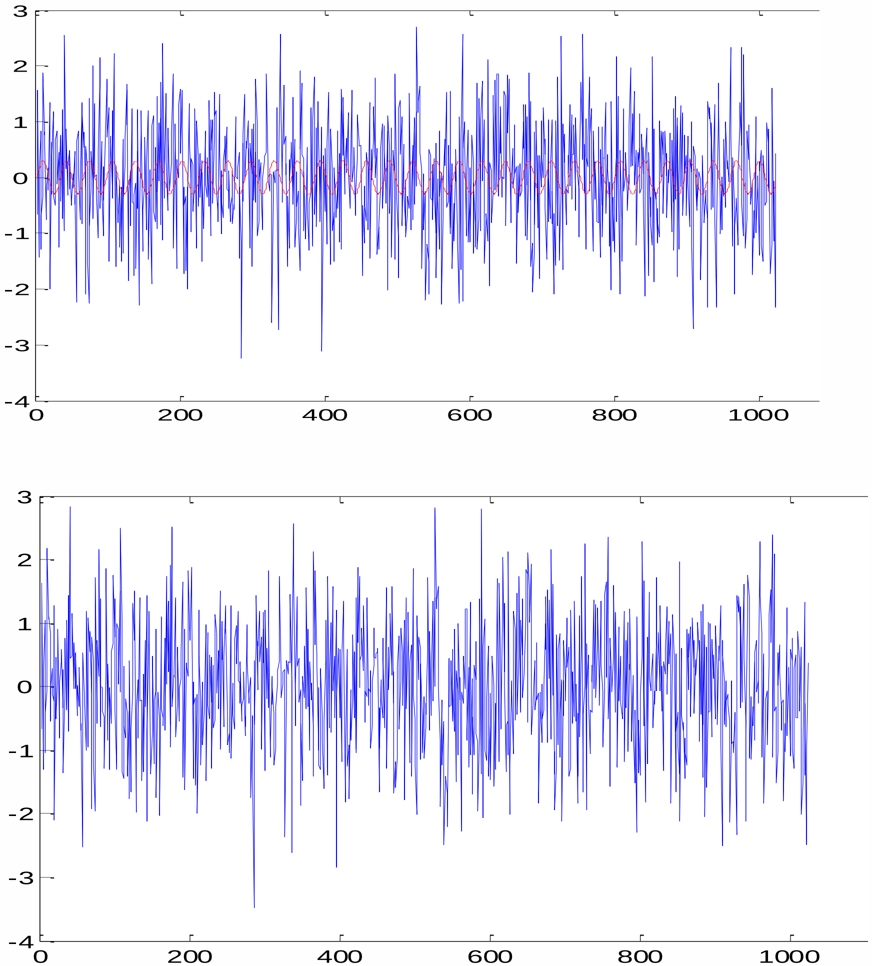
Clutter + Target $\rightarrow$ w+x

Probability of Detection P_d: Probability to correctly declare a target
Probability of False Alarm P_{fa}: Probability to declare a target when it is not present
Probability of Miss P_m: Probability to miss a target while it is present $P_m=1-P_d$

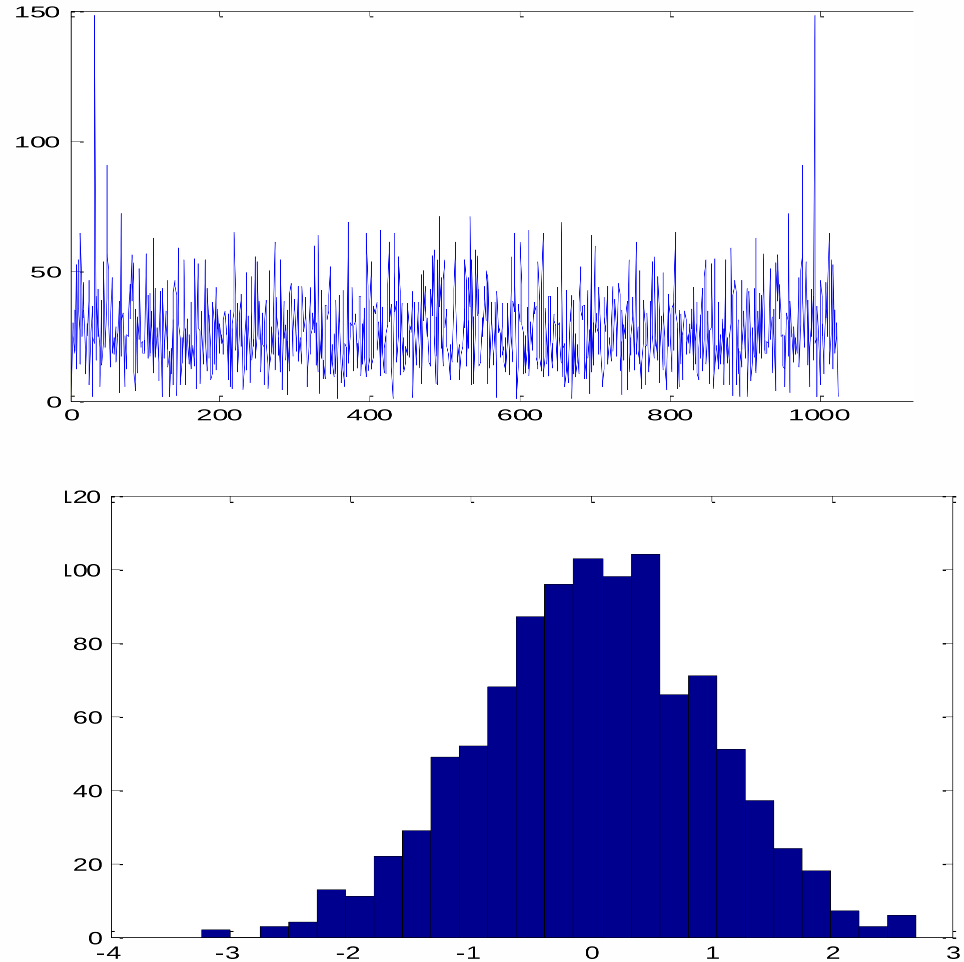




Signals and Statistics

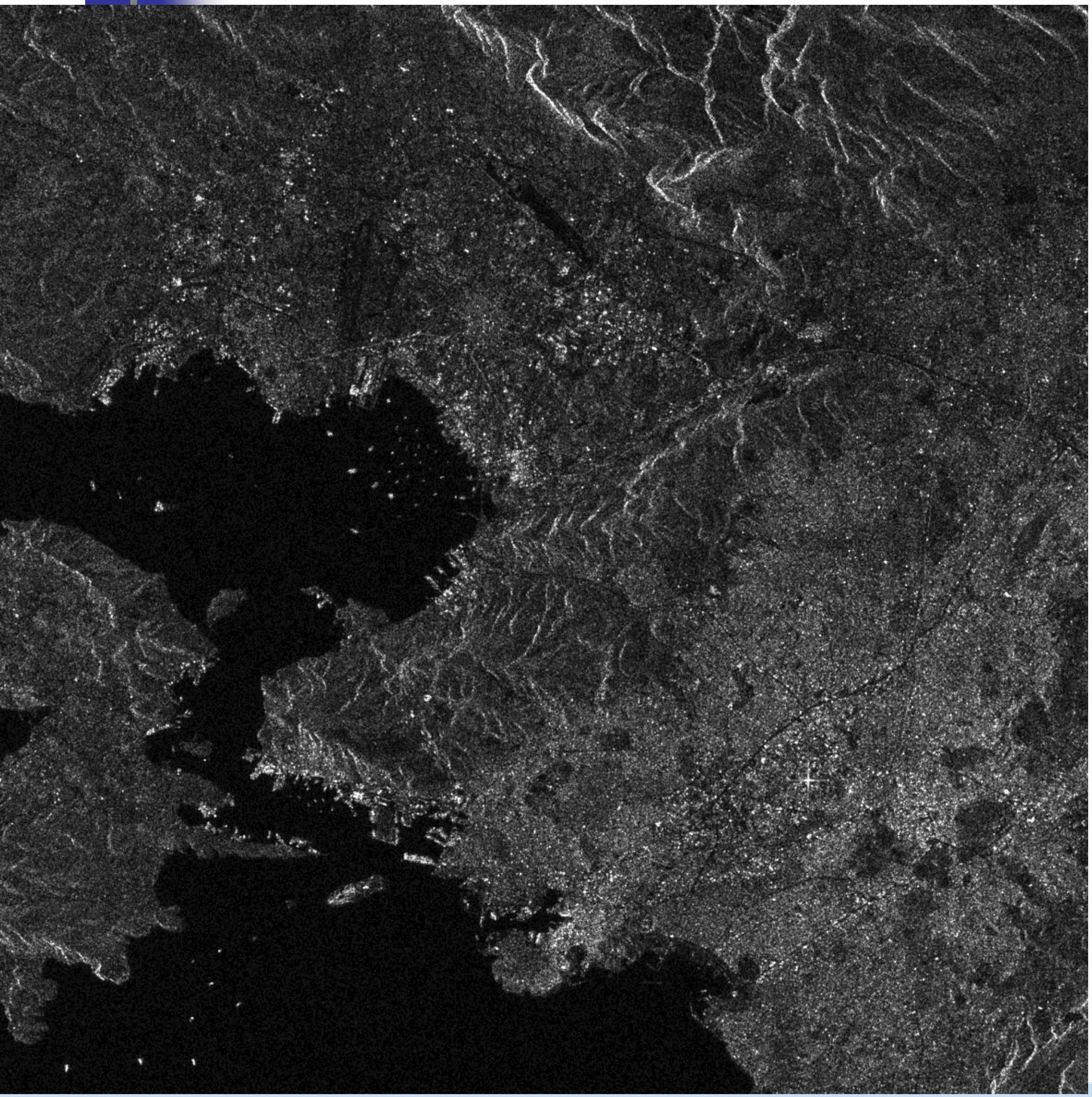


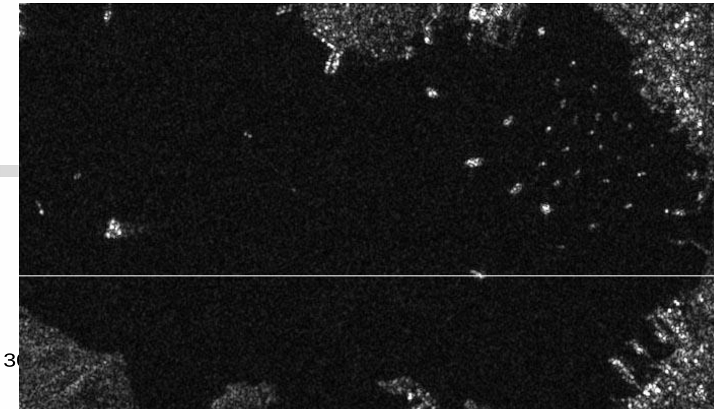
Signal+noise

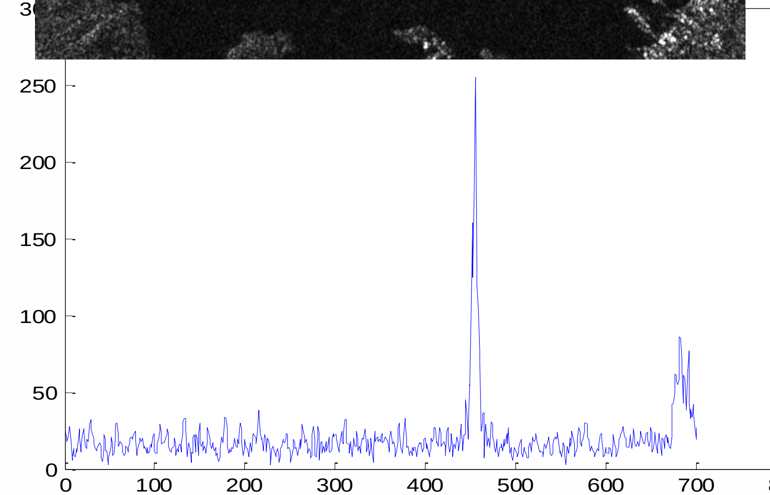


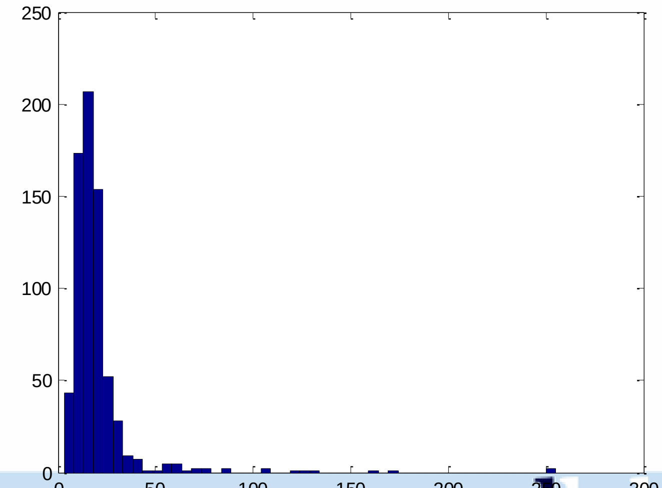
Spectrum and histogram

Images and Statistics

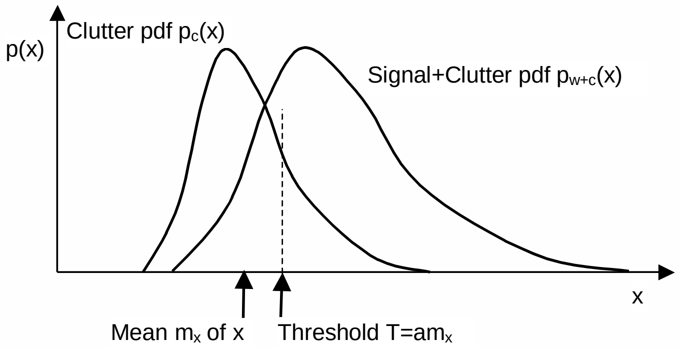








Decision based on the Likelihood Ratio



$$P_{fa} = \int_T^\infty p_x(x|H_0)dx$$

$$P_d = \int_T^\infty p_x(x|H_1)dx$$

Pattern Recognition

The Neyman-Pearson decision rule:

Choose the threshold T so that P_d is maximized, subject to $P_{fa} \leq a$

It is an optimization problem the solution of which leads to the decision rule

$$\frac{p_x(x|H_1)}{p_x(x|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

Case of $p(x|H_0)$ Gaussian

$$P_{fa} = \int_T^\infty p_x(x|H_0)dx \Leftrightarrow$$

$$\Leftrightarrow 1 - \int_T^\infty p_x(x|H_0)dx = erf(T) = 1 - P_{fa} \Leftrightarrow$$

$$\Leftrightarrow T = erf^{-1}(1 - P_{fa})$$

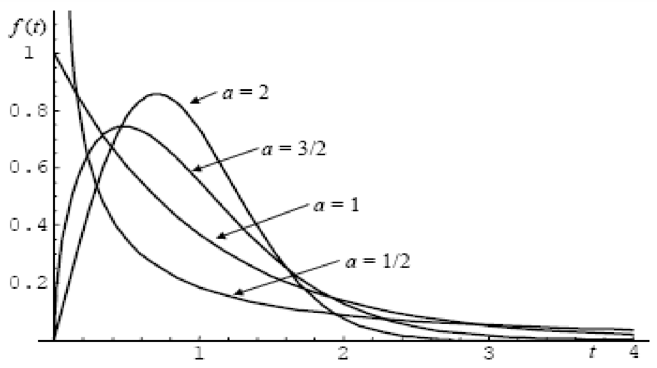
Estimation of the mean

Clutter and Target Statistics

Weibull pdf

$$f(x)=\left\{\frac{a}{c}\left(\frac{x}{c}\right)^{a-1}\exp\left[-\left(\frac{x}{c}\right)^{a}\right]\right\}$$

x,c,a >0



$$F(x)=1-\exp\left[-\left(\frac{x}{c}\right)^{a}\right]$$

For a=2 → Rayleigh pdf

χ^2 distribution $s=x_1^2+x_2^2+\ldots+x_n^2$

$$p(s)=\frac{1}{2^{n/2}\Gamma(n/2)}s^{n/2-1}\exp(-s/2)\quad s\geq 0$$

Generalized Gamma

$$p(x)=\frac{b}{s\Gamma(\nu)}\left(\frac{x}{s}\right)^{b\nu-1}\exp\left(-\left(\frac{x}{s}\right)^{b}\right)\quad s,x,b,\nu\geq 0$$

The K-distribution is a compound statistical model i.e.

$$p(x)=\int_0^\infty p(x/y)p(y)dy=\frac{4c^{m+1}}{\Gamma(m)}x^m K_{m-1}(2cx)$$

The 'speckle' x is Rayleigh distributed with mean y, which is Gamma distributed (b=2).
$K_m(x)$ is the modified bessel function of the third kind and order m, while c is the scale parameter.

High Resolution Clutter

The generalized compound probability density function (GC-pdf) is presented for modeling high resolution radar clutter. In particular, the model is used to describe deviatio of the speckle component from the Rayleigh to Weibull or other pdfs with longer tails. The GC-pdf is formed using th generalized gamma (GΓ) pdf to describe both the speckle a the modulation component of the radar clutter. The propos model is analyzed and thermal noise is incorporated into it. validation of the GC-pdf with real data is carried out empl the statistical moments as well as goodness-of-fit tests. A lar variety of experimental data is used for this purpose. The C outperforms the K-pdf in modeling high resolution radar cl and reveals its structural characteristics.

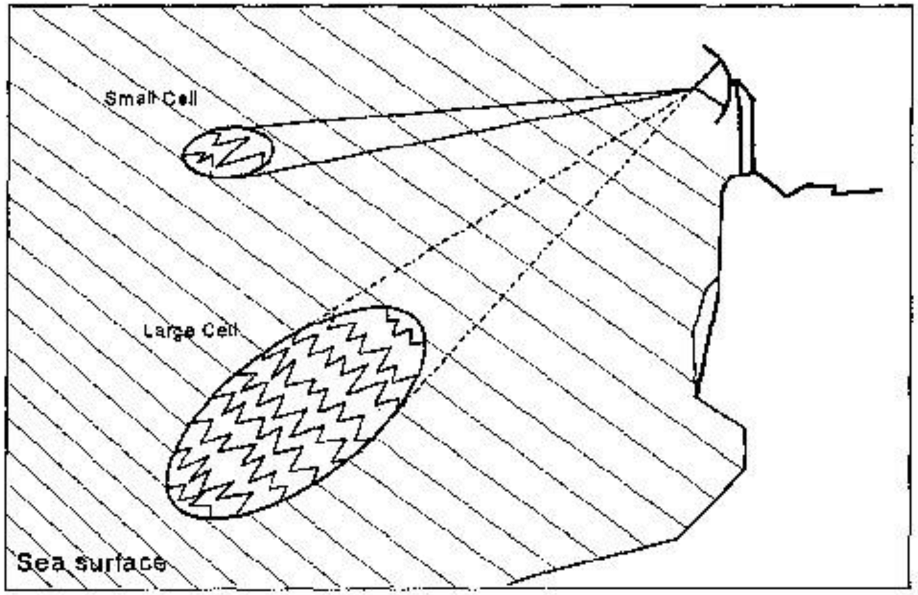


Fig. 1. Size of resolution cell determines number of scatterers contained into it. In large resolution cell there are many scatterers, CLT applies and results to Gaussian statistics. In small resolution cell only few scatterers are present and pdf has longer tail.

GC-pdf modeling

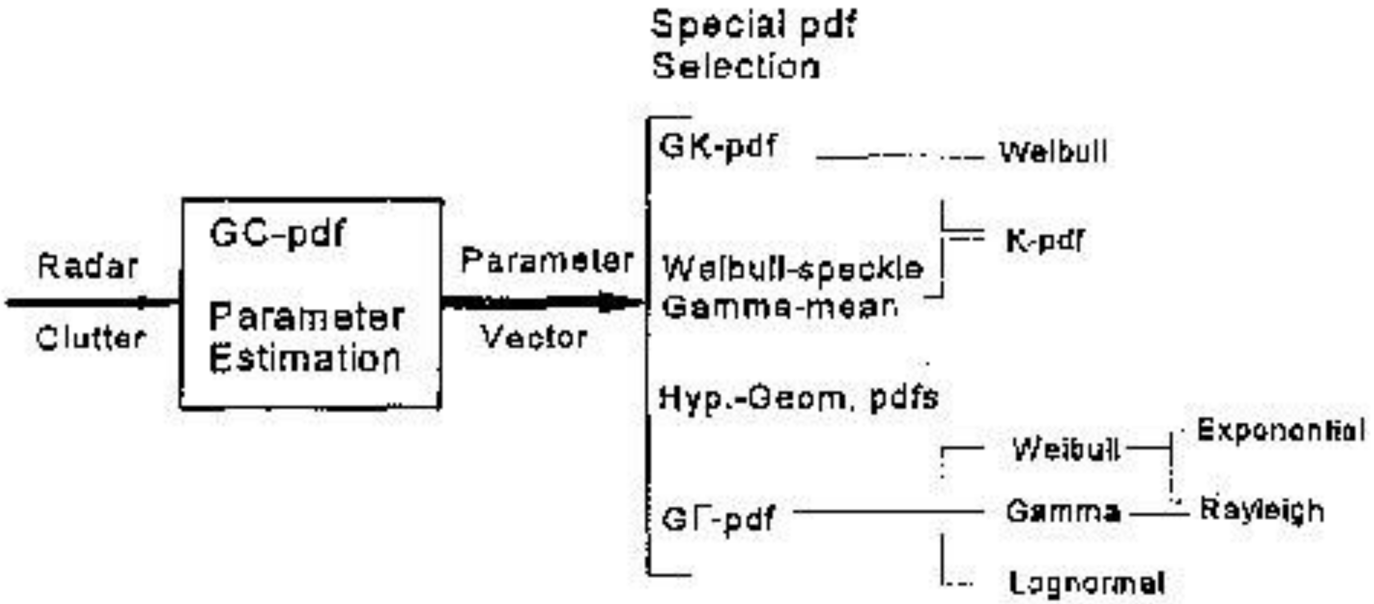
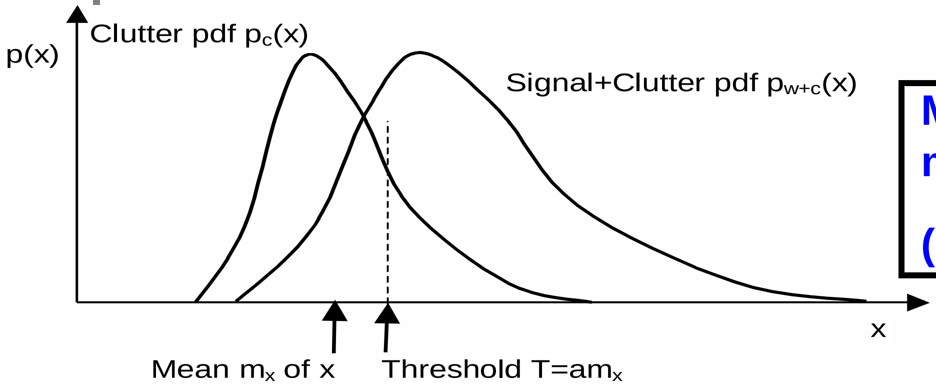


Fig. 2. GC-pdf gives as special cases almost all statistical models used for radar clutter modeling. Starting from GC-pdf, parameter vector will probably result in one of these models.

Estimation of the mean

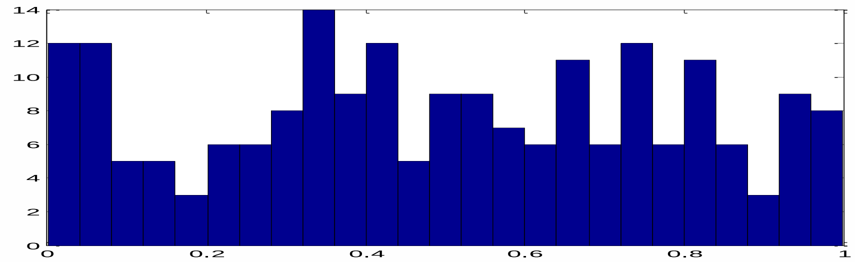
$$P_{fa} = \int_T^{\infty} p_x(x \mid H_0)\,dx$$

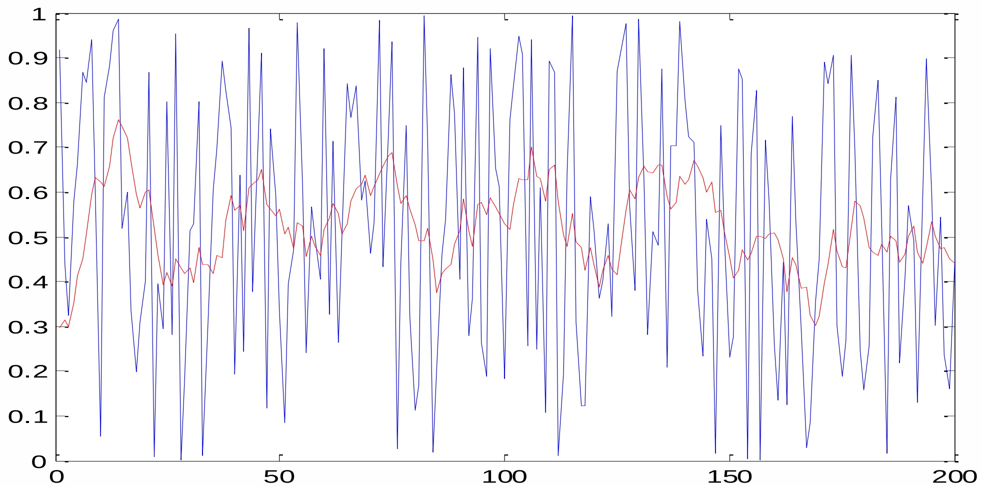


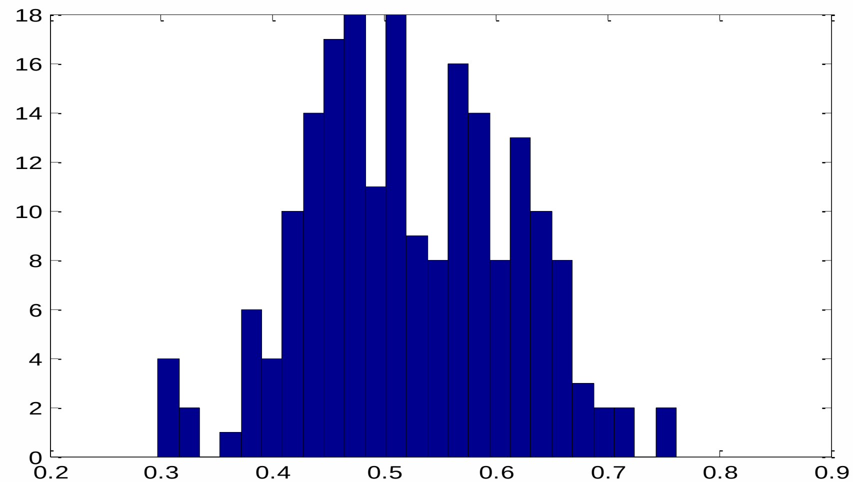
Mean estimation using the averager: $m_x=[x(1)+x(2)+\ldots+x(n)]/n$

(is it an optimal estimator?) n=10

Uniform distribution with mean 0.5

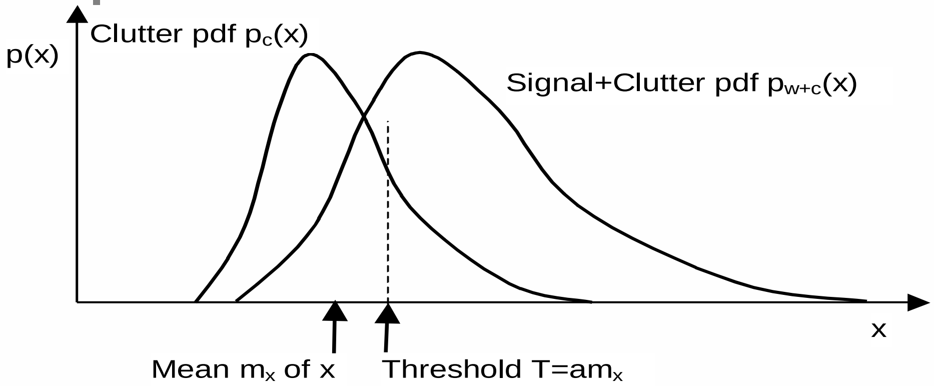






Estimation of the mean

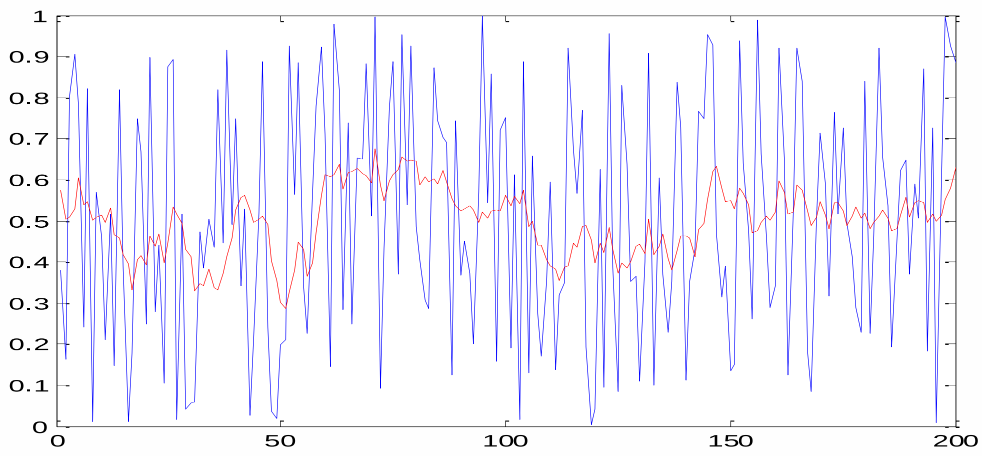
$$P_{fa} = \int_T^{\infty} p_x(x \mid H_0)dx$$

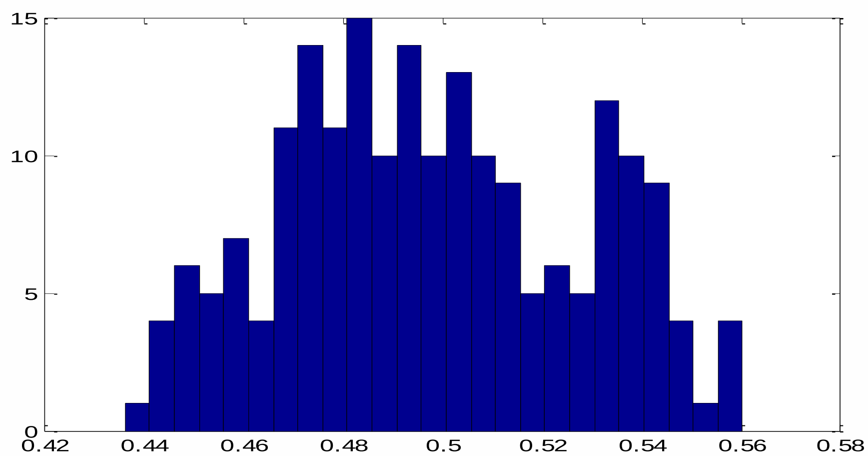


Mean estimation using the averager: $m_x=[x(1)+x(2)+\ldots+x(n)]/n$

Optimal estimator:
For the same number of variables the smallest variance in the estimation of the required parameter.

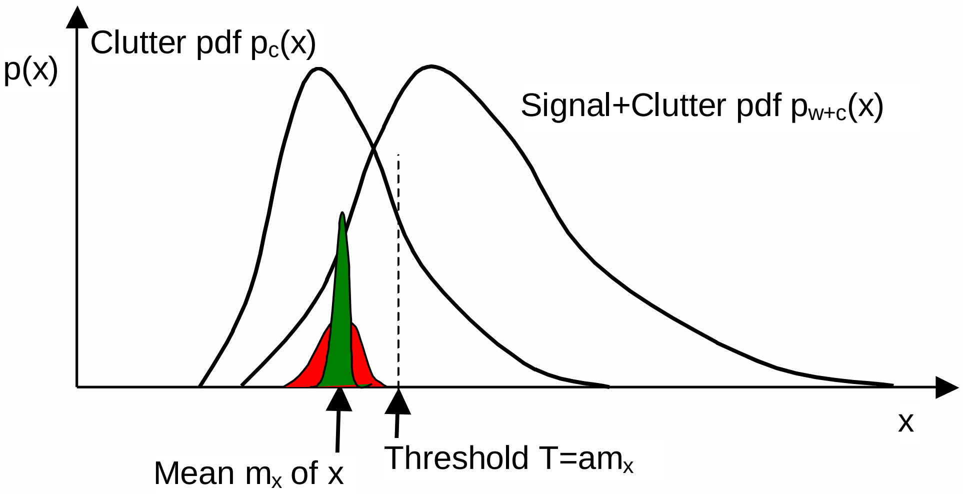
n=50





Estimation of the mean and P_{fa} evaluation

$$P_{fa} = P[x_0 > az] = \int_{T=az}^{\infty} p_x(x \mid H_0)dx$$



Since the threshold T=am$_x$=az is not fixed, but its is actually a random variable which follows a new pdf (?),

We have to determine (steps)

1. **A way to find an optimal estimator.**
2. **To find the pdf of the estimate**
3. **To modify the expression for P_{fa} and P_d.**

$$P_{fa} = P[x_0 > az] = \int_{T=az}^{\infty} p_x(x \mid H_0)dx$$

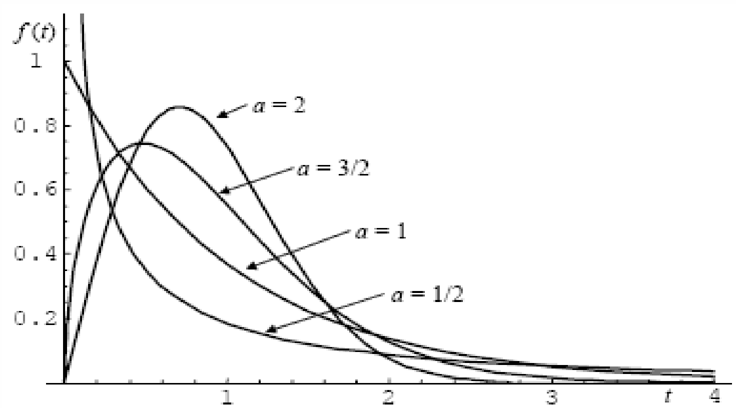
$$= \int_{0}^{\infty} P[x_0 > ax] p_z(x)dx$$

Case of Weibull - statistics

$$f(x) = \left\{ \frac{a}{c} \left(\frac{x}{c} \right)^{a-1} \exp\left[-\left(\frac{x}{c} \right)^{a} \right] \right\}$$

$$F(x) = 1 - \exp\left[-\left(\frac{x}{c} \right)^{a} \right]$$

$$E[x^r] = c^r \Gamma\left[\left(\frac{2r}{a} \right) + 1 \right]$$



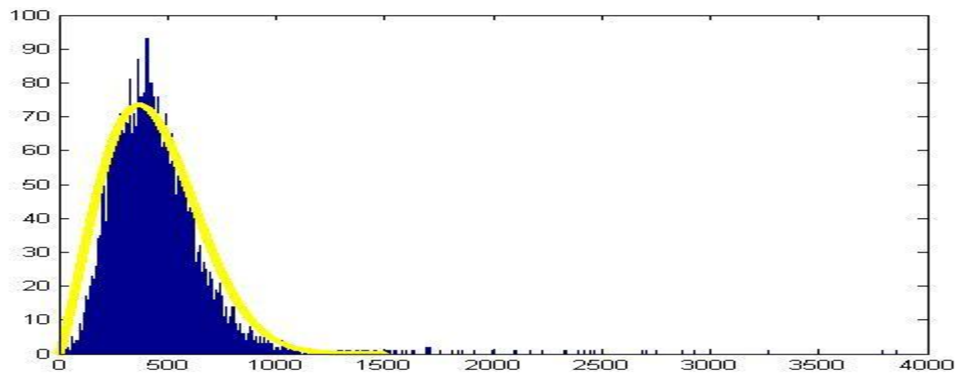
Fits well to sea statistics.

Mathematically it is well described.

With statistical fit tests or using moment matching techniques we can certify (or not) the validity of the distribution.

However, estimation of the parameters at each point of the signal space we need an optimal estimator.

Parameter estimation with statistical fit-tests



Histogram with blue (experimental pdf) for a squared Sea region on a VV-SAR image with scale parameter c=492,1 and και shape parameter a=2,16. Parameter estimation for this region (many points) was carried out by the MATLAB command “**Wblfit”** and confidence interval 99%.

Maximum Likelihood Estimation

The General Principle

Suppose that we separate a collection of samples according to class, so that we have c data sets, $\mathcal{D}_1, \ldots, \mathcal{D}_c$, with the samples in $\mathcal{D}_j$ having been drawn independently according to the probability law $p(\mathbf{x}|\omega_j)$. We say such samples are *i.i.d.*—independent and identically distributed random variables. We assume that $p(\mathbf{x}|\omega_j)$ has a known parametric form, and is therefore determined uniquely by the value of a parameter vector $\boldsymbol{\theta}_j$. For example, we might have $p(\mathbf{x}|\omega_j) \sim N(\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)$, where $\boldsymbol{\theta}_j$ consists of the components of $\boldsymbol{\mu}_j$ and $\boldsymbol{\Sigma}_j$. To show the dependence of $p(\mathbf{x}|\omega_j)$ on $\boldsymbol{\theta}_j$ explicitly, we write $p(\mathbf{x}|\omega_j)$ as $p(\mathbf{x}|\omega_j, \boldsymbol{\theta}_j)$. Our problem is to use the information provided by the training samples to obtain good estimates for the unknown parameter vectors $\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_c$ associated with each category.

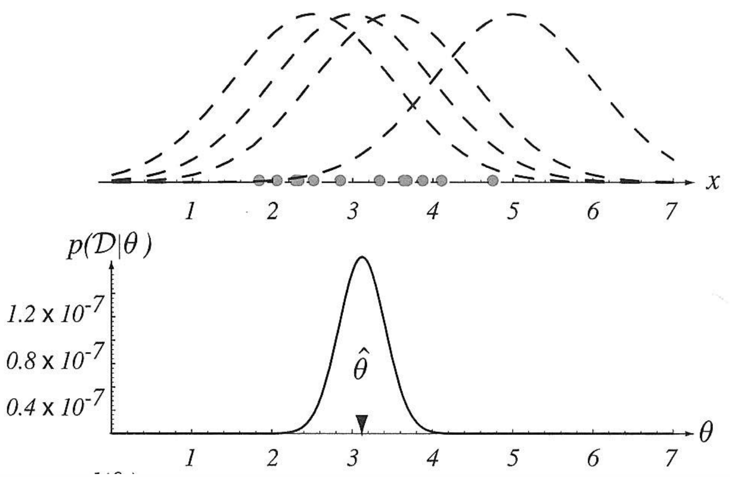
Maximum Likelihood Estimation

Suppose that $\mathcal{D}$ contains n samples, $\mathbf{x}_1, \ldots, \mathbf{x}_n$. Then, because the samples were drawn independently, we have

$$p(\mathcal{D}|\boldsymbol{\theta}) = \prod_{k=1}^{n} p(\mathbf{x}_k|\boldsymbol{\theta}). \qquad (1)$$

Recall from Chapter 2 that, viewed as a function of $\boldsymbol{\theta}$, $p(\mathcal{D}|\boldsymbol{\theta})$ is called the *likelihood* of $\boldsymbol{\theta}$ with respect to the set of samples. The *maximum-likelihood estimate* of $\boldsymbol{\theta}$ is, by definition, the value $\hat{\boldsymbol{\theta}}$ that maximizes $p(\mathcal{D}|\boldsymbol{\theta})$. Intuitively, this estimate corresponds to the value of $\boldsymbol{\theta}$ that in some sense best agrees with or supports the actually observed training samples (Fig. 3.1).

For analytical purposes, it is usually easier to work with the logarithm of the likelihood than with the likelihood itself. Because the logarithm is monotonically increasing, the $\hat{\boldsymbol{\theta}}$ that maximizes the log-likelihood also maximizes the likelihood. If $p(\mathcal{D}|\boldsymbol{\theta})$ is a well-behaved, differentiable function of $\boldsymbol{\theta}$, $\hat{\boldsymbol{\theta}}$ can be found by the standard methods of differential calculus. If the number of parameters to be estimated is



Maximum Likelihood Estimation

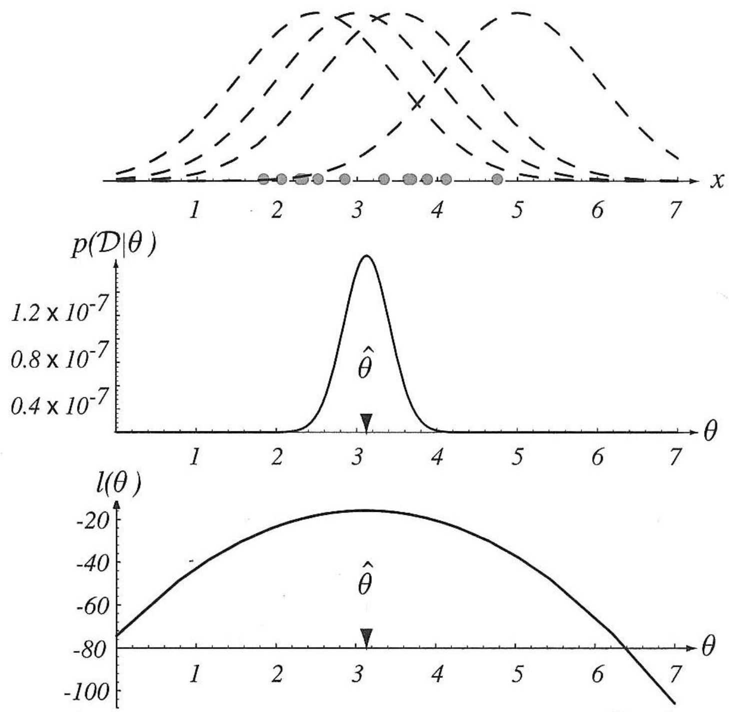


FIGURE 3.1. The top graph shows several training points in one dimension, known or assumed to be drawn from a Gaussian of a particular variance, but unknown mean. Four of the infinite number of candidate source distributions are shown in dashed lines. The middle figure shows the likelihood $p(\mathcal{D}|\theta)$ as a function of the mean. If we had a very large number of training points, this likelihood would be very narrow. The value that maximizes the likelihood is marked $\hat{\theta}$; it also maximizes the logarithm of the likelihood—that is, the log-likelihood $l(\theta)$, shown at the bottom. Note that even though they look similar, the likelihood $p(\mathcal{D}|\theta)$ is shown as a function of θ whereas the conditional density $p(x|\theta)$ is shown as a function of x. Furthermore, as a function of θ, the likelihood $p(\mathcal{D}|\theta)$ is not a probability density function and its area has no significance.

Maximum Likelihood Estimation

LOG-LIKELIHOOD

We define $l(\boldsymbol{\theta})$ as the *log-likelihood* function*

$$l(\boldsymbol{\theta}) \equiv \ln p(\mathcal{D}|\boldsymbol{\theta}). \tag{3}$$

We can then write our solution formally as the argument $\boldsymbol{\theta}$ that maximizes the log-likelihood, that is,

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} l(\boldsymbol{\theta}), \tag{4}$$

where the dependence on the data set $\mathcal{D}$ is implicit. Thus we have from Eq. 1

$$l(\boldsymbol{\theta}) = \sum_{k=1}^{n} \ln p(\mathbf{x}_k|\boldsymbol{\theta}) \tag{5}$$

and

$$\nabla_{\boldsymbol{\theta}} l = \sum_{k=1}^{n} \nabla_{\boldsymbol{\theta}} \ln p(\mathbf{x}_k|\boldsymbol{\theta}). \tag{6}$$

Thus, a set of necessary conditions for the maximum-likelihood estimate for $\boldsymbol{\theta}$ can be obtained from the set of p equations

$$\nabla_{\boldsymbol{\theta}} l = \mathbf{0}. \tag{7}$$

Sum of Two Random Variables

Let W be a random variable equal to the sum of two independent random variables X and Y:

$$W = X + Y \qquad (4.6\text{-}1)$$

This is a very practical problem because X might represent a random signal voltage and Y could represent random noise at some instant in time. The sum W would represent a signal-plus-noise voltage available to some receiver.

The probability distribution function we seek is defined by

$$F_W(w) = P\{W \leq w\} = P\{X + Y \leq w\} \qquad (4.6\text{-}2)$$

Figure 4.6-1 illustrates the region in the xy plane where $x + y \leq w$. Now from (4.3-5f), the probability corresponding to an elemental area $dx\,dy$ in the xy plane located at the point (x, y) is $f_{X,Y}(x, y)\,dx\,dy$. If we sum all such probabilities over the region where $x + y \leq w$ we will obtain $F_W(w)$. Thus

$$F_W(w) = \int_{-\infty}^{\infty} \int_{x=-\infty}^{w-y} f_{X,Y}(x, y)\,dx\,dy \qquad (4.6\text{-}3)$$

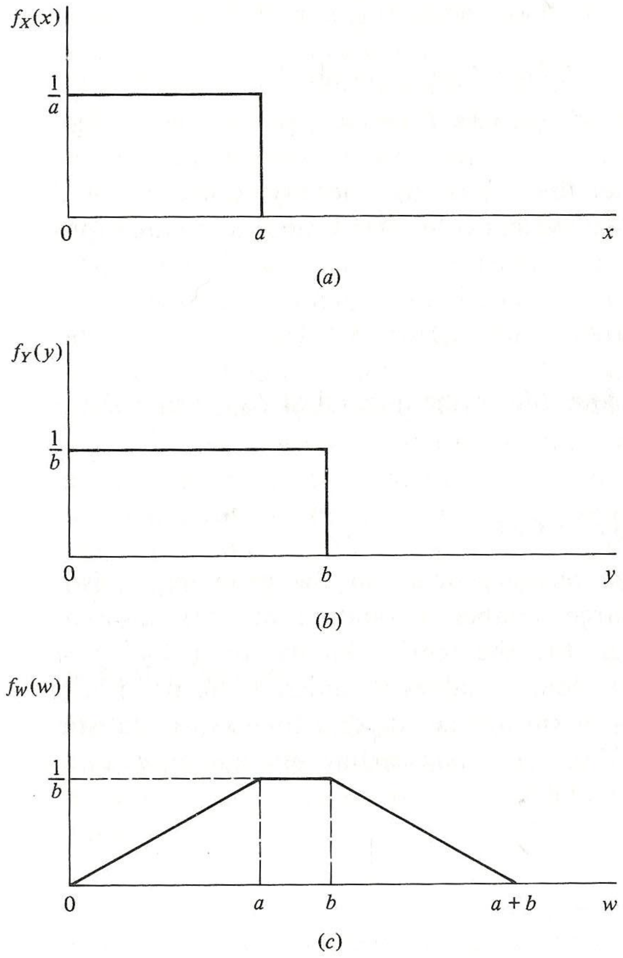
and, after using (4.5-4):

$$F_W(w) = \int_{-\infty}^{\infty} f_Y(y) \int_{x=-\infty}^{w-y} f_X(x)\,dx\,dy \qquad (4.6\text{-}4)$$

By differentiating (4.6-4), using Leibniz's rule, we get the desired density function

$$f_W(w) = \int_{-\infty}^{\infty} f_Y(y) f_X(w - y)\,dy \qquad (4.6\text{-}5)$$

This expression is recognized as a convolution integral. Consequently, we have shown that *the density function of the sum of two statistically independent random variables is the convolution of their individual density functions.*



Sum of Two Random Variables

Let W be a random variable equal to the sum of two independent random variables X and Y:

$$W = X + Y \tag{4.6-1}$$

This is a very practical problem because X might represent a random signal voltage and Y could represent random noise at some instant in time. The sum W would represent a signal-plus-noise voltage available to some receiver.

FIGURE 4.6-2
Two density functions (a) and (b) and their convolution (c).

*Sum of Several Random Variables

When the sum of Y of N independent random variables $X_1, X_2, \ldots, X_N$ is to be considered, we may extend the above analysis for two random variables. Let $Y_1 = X_1 + X_2$. Then we know from the preceding work that $f_{Y_1}(y_1) = f_{X_2}(x_2) * f_{X_1}(x_1)$.† Next, we know that X_3 will be independent of $Y_1 = X_1 + X_2$ because X_3 is independent of both X_1 and X_2. Thus, by applying (4.6-5) to the two variables X_3 and Y_1 to find the density function of $Y_2 = X_3 + Y_1$, we get

$$f_{Y_2=X_1+X_2+X_3}(y_2) = f_{X_3}(x_3) * f_{Y_1=X_1+X_2}(y_1)$$
$$= f_{X_3}(x_3) * f_{X_2}(x_2) * f_{X_1}(x_1) \qquad (4.6\text{-}6)$$

By continuing the process we find that the density function of $Y = X_1 + X_2 + \cdots + X_N$ is the $(N-1)$-fold convolution of the N individual density functions:

$$f_Y(y) = f_{X_N}(x_N) * f_{X_{N-1}}(x_{N-1}) * \cdots * f_{X_1}(x_1) \qquad (4.6\text{-}7)$$

The distribution function of Y is found from the integral of $f_Y(y)$ using (2.3-6c).

*4.7 CENTRAL LIMIT THEOREM

Broadly defined, the *central limit theorem* says that the probability distribution function of the sum of a large number of random variables approaches a gaussian distribution. Although the theorem is known to apply to some cases of statistically *dependent* random variables (Cramér, 1946, p. 219), most applications, and the largest body of knowledge, are directed toward statistically independent random variables. Thus, in all succeeding discussions we assume statistically independent random variables.

3.4 TRANSFORMATIONS OF A RANDOM VARIABLE

Quite often one may wish to transform (change) one random variable X into a new random variable Y by means of a transformation

$$Y = T(X) \tag{3.4-1}$$

$$f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|$$

ML-Parameter Estimation – Weibull pdf (1/3)

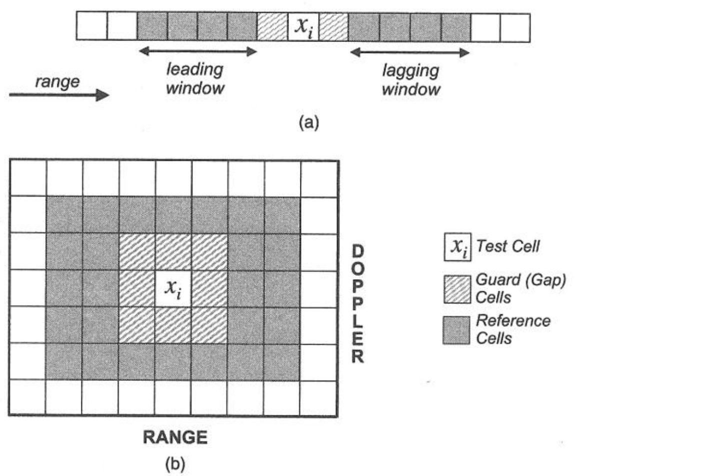


Figure 7.3 CFAR windows. (*a*) One-dimensional window for range-only processor. (*b*) Two-dimensional window for range-Doppler processor.

$$f(x) = \left\{ \frac{c}{2\rho} \left(\frac{x}{\rho} \right)^{(c/2)-1} \exp\left[-\left(\frac{x}{\rho} \right)^{c/2} \right] \right\}$$

IV. OPTIMAL DETECTION IN WEIBULL CLUTTER

A. OW Mean Estimator Derivation

Optimal mean estimation of the clutter power, in the maximum likelihood (ML) sense, is carried out in case of uniform Weibull statistics. The skewness parameter c is assumed to be known. Given N independent and identically distributed random variables, obeying (7), the ML statistic for estimating ρ is as follows:

$$l(\rho/x_1,\ldots,x_N) = \left(\frac{c}{2}\right)^N \frac{1}{\rho^{Nc/2}} \prod_{i=1}^{N} x_i^{((c/2)-1)} \times \exp\left[-\frac{1}{\rho^{c/2}} \sum_{i=1}^{N} x_i^{c/2}\right]. \qquad (19)$$

ML-Parameter Estimation – Weibull pdf (2/3)

Since l is a positive quantity, taking the natural logarithm of both sides in (19) we have

$$L(\rho/x_1,\ldots,x_N) = \ln\left(\frac{c}{2}\right)^N - \ln\rho^{Nc/2} + \sum_{i=1}^{N}\ln x_i^{((c/2)-1)} - \frac{1}{\rho^{c/2}}\sum_{i=1}^{N} x_i^{c/2}. \quad (20)$$

Taking the first derivative of L with respect to ρ and equating with zero, the MLE of ρ is obtained

$$\frac{\partial L}{\partial \rho} = -\frac{Nc}{2\rho} + \frac{c/2}{\rho^{(c/2)+1}}\sum_{i=1}^{N} x_i^{c/2} = 0$$

$$\Rightarrow \hat{\rho} = \left[\frac{1}{N}\sum_{i=1}^{N} x_i^{c/2}\right]^{2/c}. \quad (21)$$

ML-Parameter Estimation – Weibull pdf (3/3)

Substituting $\hat{\rho}$ into (11), the MLE of the mean power of the Weibull clutter (OW estimator) is obtained:

$$\hat{m} = \langle \text{clutter power} \rangle = E[x]$$

$$= \hat{\rho}\Gamma\left[\left(\frac{2}{c}\right) + 1\right]$$

$$= \left[\frac{1}{N}\sum_{i=1}^{N} x_i^{c/2}\right]^{2/c} \Gamma\left[\left(\frac{2}{c}\right) + 1\right] \qquad (22)$$

The effectiveness of the derived OW estimator can be tested by evaluating the bias and the variance for the estimate $\hat{m}$.

The pdf of the estimator statistic (1/3)

$$f(x) = \left\{ \frac{c}{2\rho} \left(\frac{x}{\rho} \right)^{(c/2)-1} \exp\left[-\left(\frac{x}{\rho} \right)^{c/2} \right] \right\}$$

Weibull pdf

The pdf of the statistic z at the output of the OW mean estimator is evaluated here. The statistic z is repeated in the following for convenience:

$$z = \hat{m} = \left[\frac{1}{N} \sum_{i=1}^{N} x_i^{c/2} \right]^{2/c} \Gamma\left[\left(\frac{2}{c} \right) + 1 \right]. \tag{48}$$

The analytic expression of $p_z(x)$ is derived in steps as follows.

Step 1 Let $y = g(x) = x^{c/2}$ and x is Weibull distributed (see (7)). The pdf $p_y(y)$ is given by [22]

$$p_y(y) = \frac{f_x(x)}{|g'(x)|} \tag{49}$$

with x replaced by $y^{2/c}$, (49) results in

$$p_y(y) = \frac{1}{\rho^{c/2}} \exp\left(-\frac{y}{\rho^{c/2}} \right). \tag{50}$$

The pdf of the estimator statistic (2/3)

$$z = \hat{m} = \left[\frac{1}{N}\sum_{i=1}^{N} x_i^{c/2}\right]^{2/c} \Gamma\left[\left(\frac{2}{c}\right)+1\right].$$

Step 2 Let $L = \sum_{i=1}^{N} y_i$ and y_i distributed according to (50). If y_i are independent and identically distributed, the moment-generating function of L, M_L, is equal to the product of the individual moment-generating functions, M_{y_i}, [23]:

$$M_L(t) = \prod_{i=1}^{N} M_{y_i}(t). \tag{51}$$

The moment-generating function $M_y = 1/(1-\rho^{c/2}t)$ and thus $M_L = 1/(1-\rho^{c/2}t)^N$. Consequently,

$$p_L(L) = \frac{1}{(\rho^{c/2})^N \Gamma(N)} L^{N-1} \exp\left[-\frac{L}{\rho^{c/2}}\right]. \tag{52}$$

Step 3 For $L_1 = L/N$ according to Step 1, the pdf $p_{L_1}(L_1)$ is found to be

$$p_{L_1}(L_1) = \frac{N^N}{(\rho^{c/2})^N \Gamma(N)} L_1^{N-1} \exp\left[-\frac{N L_1}{\rho^{c/2}}\right]. \tag{53}$$

The pdf of the estimator statistic (3/3)

$$z = \hat{m} = \left[\frac{1}{N}\sum_{i=1}^{N} x_i^{c/2}\right]^{2/c} \Gamma\left[\left(\frac{2}{c}\right)+1\right].$$

Step 4 Letting $Q = L_1^{2/c}$ in the same way as previously we have

$$p_Q(Q) = \frac{N^N}{(\rho^{c/2})^N \Gamma(N)} \frac{c}{2} Q^{(c/2)-1} Q^{c/2(N-1)} \times \exp\left[-N\left(\frac{Q}{\rho}\right)^{c/2}\right]. \quad (54)$$

Step 5 Finally, letting $z = Q\xi$ where $\xi = \Gamma[(2/c)+1]$ we have in the same way:

$$p_z(z) = \frac{N^N}{(\rho^{c/2})^N} \frac{1}{\xi\Gamma(N)} \frac{c}{2} \left(\frac{z}{\xi}\right)^{(c/2)-1} \left(\frac{z}{\xi}\right)^{c/2(N-1)} \times \exp\left[-N\left(\frac{z}{\xi\rho}\right)^{c/2}\right] \quad (55)$$

which is the final expression which we wanted to derive.

OW-CFAR Performance

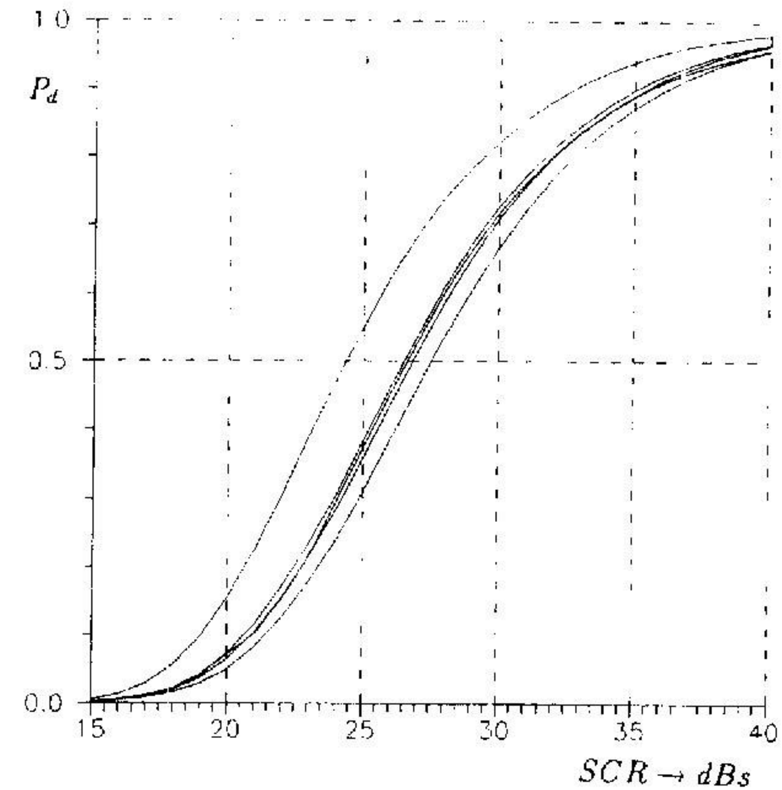


Fig. 4. Simulated P_d versus SCR (Swerling II target). From left to right: Ideal, OW, MEMO, CA and OS-CFAR detector.

The P_{fa} for the proposed OW detector in case of uniform clutter, is derived as follows:

$$P_{fa} = P[Y_0 \geq Tz] = \int_0^{\infty} P[Y_0 \geq Tx] p_z(x)\,dx \qquad (39)$$

where z is the statistic at the output of the OW detector, $p_z(z)$ the pdf of this statistic (derived in Appendix A) and T the threshold multiplier. The first term in the integral of (39) can be evaluated from (8) as follows:

$$P[Y_0 > Tx] = 1 - P(Tx) = \exp\left[\left(\frac{Tx}{\rho}\right)^{c/2} \right]. \qquad (40)$$

Consequently, (39) after some manipulations becomes

$$P_{fa} = \frac{1}{\left[\frac{(T\xi)^{c/2}}{N} + 1\right]^N} \qquad (41)$$

where $\xi = \Gamma[(2/c) + 1]$. When $c = 2$ (Rayleigh pdf) the value of P_{fa} becomes

$$P_{fa} = \frac{1}{\left[\frac{T}{N} + 1\right]^N} \qquad (42)$$

Basic steps for CFAR detection

- Knowledge of the statistics of the clutter.
- Optimal statistic for mean estimation.
- Pdf of this statistic.
- Evaluation of P_{fa}.
- Threshold evaluation.
- P_d evaluation for various SCRs (Performance assessment - ROC)

$$P_{fa} = P[x_0 > az] = \int_{T=az}^{\infty} p_x(x \mid H_0)dx = \int_0^{\infty} P[x_0 > ax] p_z(x)dx$$

$$P_{fa} = \frac{1}{\left[\frac{(T\xi)^{c/2}}{N} + 1\right]^N}$$

$$P_d = P[x_1 > az] = \int_{T=az}^{\infty} p_x(x \mid H_1)dx = \int_0^{\infty} P[x_1 > ax] p_z(x)dx$$

- **Assessment using Simulation**

CFAR performance

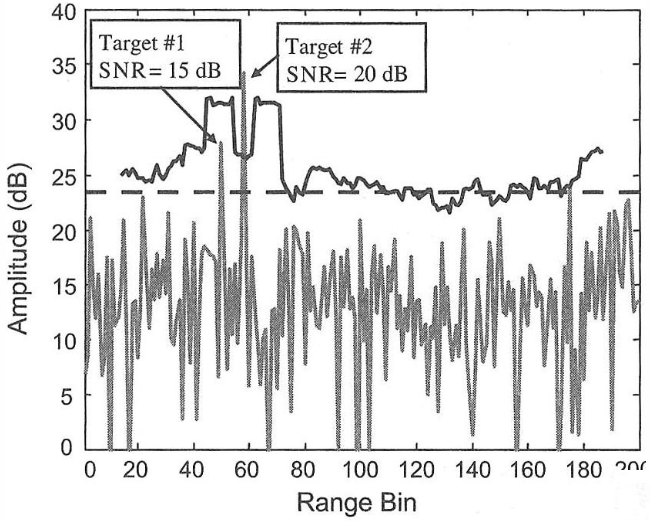


Figure 7.6 Illustration of target masking. (See text details.)

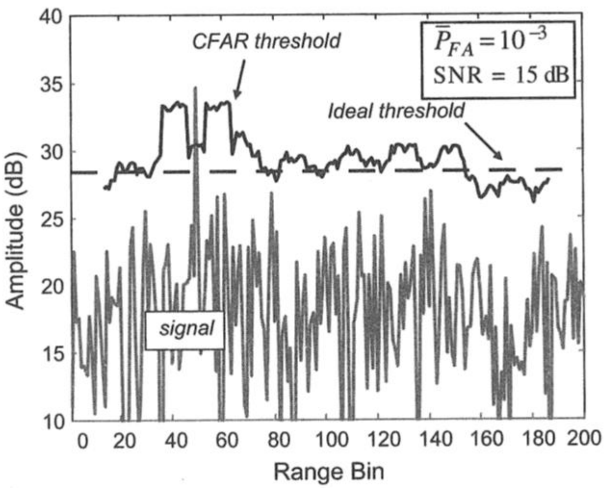


Figure 7.4 Example of cell-averaging CFAR threshold behavior.

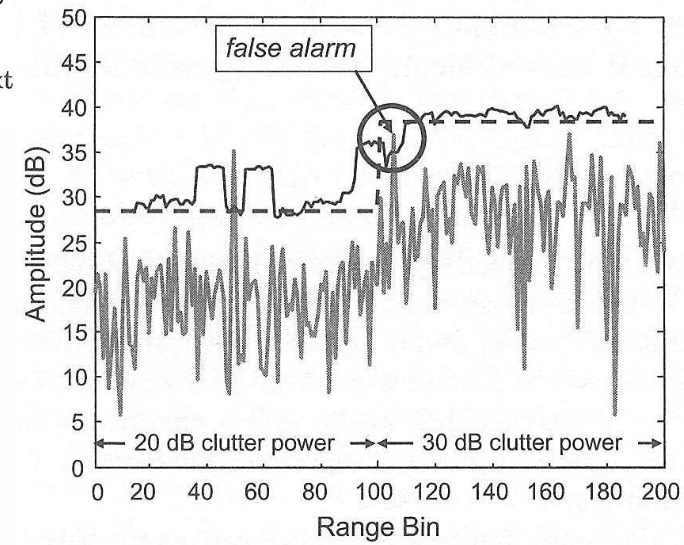


Figure 7.9 False alarms at a clutter edge.

Conclusions (1/2)

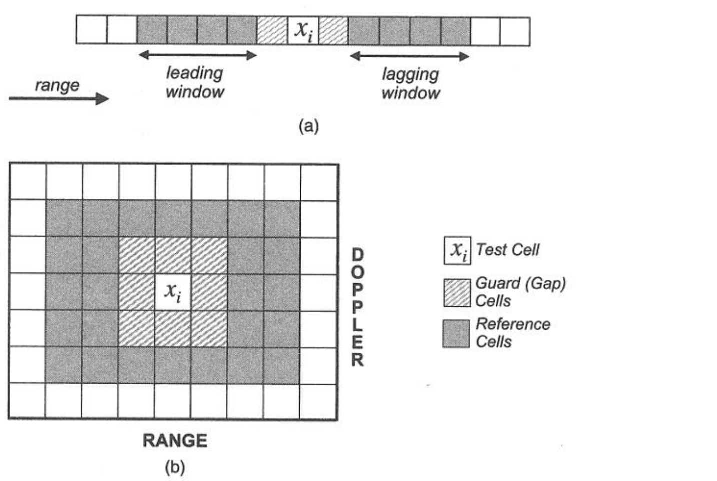
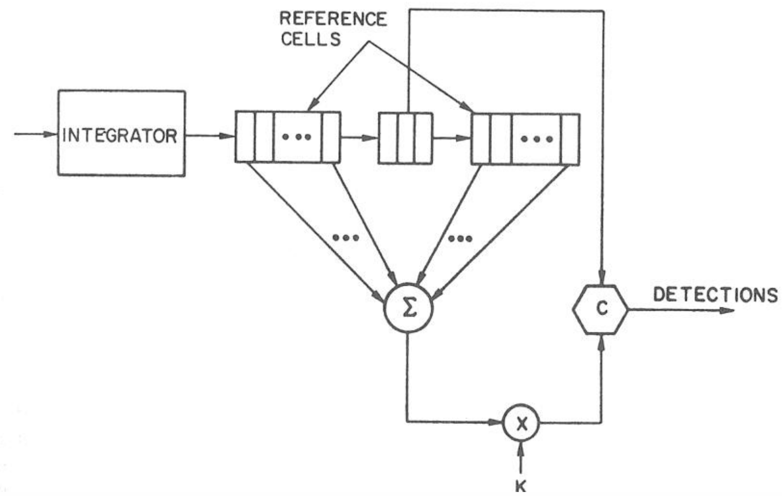


Figure 7.3 CFAR windows. (a) One-dimensional window for range-only processor. (b) Two-dimensional window for range-Doppler processor.



- For robust detectors we employ Order Statistic Estimators.
- Performance Assessment contains performance on clutter edges, and target masking.
- Clutter maps constitute a-priori knowledge for the clutter statistics of a regions. Accelerate detection.

Conclusions (2/2)

- We examined CFAR detection of point targets in specific clutter statistics and described performance assessment.
- Many other detection topics remain:

  - Distributed targets

  - Coherent detection

  - Detection using multi-channel signals
- What is detection and its quality; How this can be used in decision fusion;